Problems in Quantum Field Theory 2: Free quantum fields

2015 Belgian-Dutch-German
Graduate School of Particle Physics

V. Pelgrims

NOTE: Priority to exercises marked by a "⋆" will be given during the tutorials.

1. Show that the Dirac and the Weyl representations are equivalent.

2. Show that, for fermions, helicity and angular momentum are conserved.

3. ⋆ Let us introduce the Weyl fields

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi,$$
$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi,$$

   where ψ is a Dirac spinor. Derive the equations of motion for these fields. Show that they are decoupled in the case of a massless spinor.

4. Prove that if $\psi(x)$ is a solution of the Dirac equation, it is also a solution of the Klein-Gordon equation.

5. ⋆ Compute the commutators $[\phi(x), \phi(x')]_{t=t'}$ and $[\pi(x), \phi(x')]_{t=t'}$.

6. ⋆ Interpret the following Lagrangian densities and find the corresponding Euler-Lagrange equations

   - $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$,
   - $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$,
   - $\mathcal{L} = (\partial_\mu\phi - ieA_\mu\phi)(\partial^\mu\phi^* + ieA^\mu\phi^*) - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$,
   - $\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 - ig\bar{\psi}\gamma_5\psi\phi$.

7. Prove that the equations of motion remain unchanged if the divergence of an arbitrary field is added to the Lagrangian density.

8. ⋆ For photons, compute the commutator $[a^\lambda, a^{\lambda' +}]$ from the canonical field commutations.

9. Show, in the Feynman gauge, that

$$H = \int \frac{d^3k}{(2\pi)^3} E_k \sum_{\lambda=1}^{3} (a^{\lambda+}a^\lambda - a^{0+}a^0).$$

10. Prove that $D_F(x - x')$ is a solution of $(\Box + m^2)D = -i\delta^{(4)}(x - y)$.

11. Calculate the photon propagator $D_{\mu\nu}(k)$ in a general covariant gauge.