Problems in Quantum Field Theory 1: Classical fields

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NOTE: Priority to exercises marked by a "⋆" will be given during the tutorials.

1. How much is 1 Tesla in natural units?
2. Write the field-strength tensor $F^{\mu\nu}$ as a function of the electric field E_i and of the magnetic field B_i.
3. ⋆ What is the field of a static charge in each of the following gauges:

$$\begin{aligned} &\text{Lorenz} \quad \partial_\mu A^\mu = 0, \\ &\text{Axial} \quad n_\mu A^\mu = 0, \\ &\text{Coulomb} \quad \vec{\nabla}.\vec{A} = 0? \end{aligned}$$

4. Show that the rapidity y is additive with respect to boosts and that $y \sim v$ if $c \to \infty$.
5. ⋆ Derive the canonical equation of motion from the Hamiltonian

$$H(p,q) = \sum_i [p_i \dot{q}_i(q,p) - L(q, \dot{q}(q,p))].$$

6. ⋆ Find, for a single particle, the action which is invariant with respect to boosts and obtain the Euler-Lagrange equation, the momentum and the energy.
7. Show that, with fields, the canonical equations of motion are given by

$$\begin{aligned} \frac{\partial H}{\partial \pi} &= \dot{\varphi}, \\ \frac{\partial H}{\partial \varphi} &= \dot{\pi}. \end{aligned}$$

8. Show that $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is a total derivative.
9. Show that the current

$$j_\mu = -\frac{i}{2}(\varphi \partial_\mu \varphi^* - \varphi^* \partial_\mu \varphi)$$

is conserved if φ obeys the Klein-Gordon equation.

10. ⋆ The Lagrangian density for a massive vector field A^μ is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu.$$

Prove that the equation $\partial_\mu A^\mu = 0$ is a consequence of the equations of motion.

11. Find the canonical Hamiltonian for free scalar and spinor fields.