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Hadronic scattering amplitudes: medium-energy constraints on asymptotic behaviour

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Abstract

We consider several classes of analytic parametrisations of hadronic scattering amplitudes, and compare their predictions to all available forward data (pp, $\overline{p}p$, πp, Kp, γp, $\gamma\gamma$, Σp). Although these parametrisations are very close for $\sqrt{s} \geq 9$ GeV, it turns out that they differ markedly at low energy, where a universal pomeron term $\sim \ln^2 s$ enables one to extend the fit down to $\sqrt{s}$=4 GeV.

1 Introduction

The singularity structure of forward hadronic amplitudes is of great importance, as it controls the extrapolation of cross sections to high energies and to small x. Its study lies mostly outside the realm of perturbative QCD, except perhaps at small x and high Q^2, where there is some overlap, hence the hope to obtain some QCD-based understanding of

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these amplitudes in the near future. However, there are several tools available to treat this non perturbative domain. These are based on the theory of the analytic S matrix.

The first is to demand that hadronic amplitudes are analytic functions in the complex angular momentum J. The singularities in the complex J plane then determine the form of the asymptotic amplitudes in s at finite t. This means that one can then relate, through analyticity and crossing symmetry, the real part of the amplitude to its imaginary part. In other words, the exact knowledge of the cross section for all s is equivalent to that of the ρ parameter. In practice, there are several analytic forms which are very close for the total cross sections in a finite interval in s, but which differ markedly for the real part. Hence in this paper, we shall consider the experimental constraints on both the real and the imaginary parts. Furthermore, t-channel unitarity leads to the conclusion that these singularities should be universal, in the sense that they do not depend on the scattering hadrons[3]. This leads to factorizing amplitudes[4], for which the residue depends on the colliding hadrons, but the singularity is independent of them.

The second constraint is due to the unitarity of partial waves and polynomial boundedness of the absorptive part within the Lehmann ellipse. This leads to the celebrated Froissart-Martin bound [1], which indicates that at asymptotic energies, total cross sections cannot increase faster than $\ln^2 s$ (note that this behaviour was first proposed by W. Heisenberg in 1952 [2]). Although this is a priori a strong constraint, it turns out that the coefficient of the $\ln^2 s$ can be large: all we know is that it is bounded by $\frac{\pi}{m_\pi^2} \simeq 60$ mb (Lukaszuk-Martin [3]), hence parametrisations which asymptotically violate the Froissart bound, such as rising simple poles, may survive to present energies without violating unitarity.

Finally, the last ingredient is Regge theory. The meson trajectories can indeed be seen in a Chew-Frautschi plot, and hence their intercepts can in principle be measured directly. This leads to the conclusion that the intercepts of these trajectories are of order 0.5, that the $C = +1$ and $C = -1$ trajectories are approximately degenerate, and that they seem to be linear. We shall assume in the following that their contribution to the total cross section can be parametrized by $Y^+ s^{\alpha_+ - 1}$ and $Y^- s^{\alpha_- - 1}$.

These constraints, unfortunately, are far from providing a unique answer. As an example, the derivative relations [4] can be conceived as a source of an infinite class of analytic parametrisations satisfying the above theoretical criteria. However, it is possible to reduce this class of models to a few exemplar cases, for which the cross section, in the limit

[3]The photon is special in this context, and may have further singularities.

[4]Note however that factorization can be proven only for simple poles.

$s \to \infty$, behaves as a constant, as $\ln s$ or as $\ln^2 s$. Hence in practice, only a handful of parametrisations have been considered and constrained. These represent variations on the parametrisation proposed in [5, 6], which will be symbolically referred to as (Regge + Regge + Pomeranchuk + Heisenberg) type parametrisations – RRPH. Here both R stand for the leading reggeon terms, P stands for a constant contribution to the total cross section at asymptotic energies (the classical Pomeranchuk asymptotic limit [7]) and H stands for the asymptotically infinitely rising with energy contribution, which we take as $\ln s$ or $\ln^2 s$. Because of its popularity and simplicity, we shall also consider case E, i.e. the case of a simple pole $s^{\alpha_{\wp}-1}$with $\alpha_{\wp} > 1$.

Some of us (COMPAS) are maintaining a complete set of data for all hadronic processes, so that we are in a position to fully evaluate the various possibilities. We are using a slightly improved dataset from the one of [8]: some preliminary data on the ρ parameter have been removed, and new published data from SELEX ($\pi^- N$ and $\Sigma^- N$ at 600 GeV/c) [9] and OPAL ($\gamma\gamma$) [10] were added. We did not use the new recent data from L3 [11] on $\gamma\gamma \to hadrons$ total hadronic cross sections because unfortunately these very interesting data are still not published yet. Definitely these data, when published, will be used in the next iteration of the cross assessments.

In the past few years, and mainly because of the existence of this dataset, several advances have been made:

1. The systematic and simultaneous study, via analytic representations, of the forward data, both σ_{tot} and ρ, for pp, $\overline{p}p$, $\pi^{\pm}p$, $K^{\pm}p$, γp and $\gamma\gamma$ scattering. Such a program was initiated by the COMPAS group[12], and pursued in refs. [8, 13];

2. The general recognition that a Regge pole model [13] has a much wider range of applicability than previously expected while it was also recognized that the exchange-degenerate reggeons were not preferred by the forward scattering data [14];

3. The rediscovery [15] of former ideas [16, 17] such as a 2-component soft pomeron, with one component taking quark counting into account and the other being universal and rising with energy, or of full lifting of degeneracy for lower meson trajectories [18].

4. The impossibility to distinguish between wide ranges of analytic parametrisations when using data at $\sqrt{s} \geq 9$ GeV [8].

We want to examine in detail those conclusions, and see to which extents the models considered in [8] can be extended to lower energy, i.e. above the resonance region $\sim$ 3 GeV. A new quantitative procedure of ranking models by the quality of the fit to the current experimental data is suggested and used. In section 2, we shall concentrate on total cross sections, and propose this new ranking scheme. In section 3, we shall extend our analysis to all forward data, and see that this changes the picture considerably. In section 4, we shall comment on some models proposed recently, and which were not considered directly in the previous analysis. In section 5, we shall comment on cosmic ray data. To conclude, we shall present the possible alternatives, and analyse their respective drawbacks and advantages.

2 Fits to lower-energy total cross sections

As it will turn out, the consideration of $\rho(s)$ data results in a very constrained fit, but some of the sub-samples of data are poorly fitted to. This might be blamed on the quality and systematic errors on the forward-scattering data for $\rho(s)$. Hence the first and safest constraint must be the reproduction of $\sigma_{tot}(s)$ data only. In this case, the number of possible models that achieve a good χ^2 per degree of freedom (χ^2/dof) is quite large. To describe the different possibilities we will need some notations to classify variants, and we shall use the following:

$$\sigma^{a_{\mp}b} = \frac{1}{s}\left(\mathrm{R}^{+ab}(s) \pm \mathrm{R}^{-ab}(s) + \mathrm{P}^{ab} + \mathrm{H}^{ab}(s)\right), \tag{1}$$

where:

- $\mathrm{R}^{+ab}(s) = Y_1^{ab} \cdot (s/s_1)^{\alpha_1}$, with $s_1 = 1$ GeV^2,
- $\mathrm{R}^{-ab}(s) = Y_2^{ab} \cdot (s/s_1)^{\alpha_2}$,
- $\mathrm{P}^{ab} = sC^{ab}$ is the Pomeron simple pole at $J = 1$
- $\mathrm{H}^{ab}(s)$ stands for one of the three following possibilities :

  . a supplementary simple pole at $J = \alpha_{\wp}$, with $\alpha_{\wp} > 1$:

$$\mathrm{E}^{ab} = X^{ab}(s/s_1)^{\alpha_{\wp}};$$

. a double pole at $J = 1$:

$$L_{ab} = s(B_{ab} \ln(s/s_1) + A_{ab});$$

. a triple pole at $J = 1$:

$$L2_{ab} = s(B_{ab}(\ln^2(s/s_0) + A_{ab}).$$

In the general case, the constants C_{ab} and A_{ab} are independent and they are associated with a different behaviour in t. But at $t = 0$, as is the case for our fits, they can't be distinguished. They mix and we are left, when we consider logarithms, with just linear or quadratic forms in $\ln s$:

$$P_{ab} + L_{ab} = sB_{ab} \ln(s/s_1) + sZ_{ab}$$

and

$$P_{ab} + L2_{ab} = sB_{ab} \ln^2(s/s_0) + sZ_{ab},$$

where $Z_{ab} = C_{ab} + A_{ab}$.

In the following we will restrict ourselves to fits where s_0 is process-independent. We have also considered fits with the ratio Z_{ab}/B_{ab} kept process-independent

$$P_{ab} + L_{ab} = s\lambda_{ab}(B \ln(s/s_1) + A),$$

with $\lambda_{pp} = 1$, as well as fits to the form RRE, without any P term.

We fit to 3 pairs of reactions for particle and antiparticle: pp and $\bar{p}p$, $\pi^{\pm}p$ and $K^{\pm}p$, one reaction with particles $\Sigma^- p$ and two reactions coupled only to C=+1 trajectories: γp and $\gamma\gamma$.

The counting of parameters then goes as follows:

- one intercept, and 6 residues (*i.e.* 7 parameters) for each $C = +1$ reggeon;
- one intercept and 4 residues (*i.e.* 5 parameters) for each $C = -1$ reggeon.

Concerning the pomeron terms, unless otherwise indicated by the subscript nf, we impose factorisation of the γ cross sections: $H_{\gamma\gamma} = \delta H_{\gamma p} = \delta^2 H_{pp}$ and/or $P_{\gamma\gamma} = \delta P_{\gamma p} = \delta^2 P_{pp}$ with the same value of δ. This leads to:

- 1 parameter δ;
- 4 parameters for the constant term Z_{ab};
- 4 parameters B_{ab} + one intercept for E or one scale factor s_0 for L2;

When considering several singularities for the pomeron term, we usually treat them as independent. However, when we implement factorisation, we take the same value of δ for all singularities. This leads to:

- 9 parameters for PL;
- 10 parameters for PL2 or PE.

Furthermore, we have considered several possibilities to constrain the parameters. The following notations are attached as either superscript or subscript to the model variants in each case:

d means degenerate leading reggeon trajectories $\alpha_1 = \alpha_2$. This lowers the number of parameters by 2 units, as one has only one intercept, and one coupling for the $\Sigma^- p$ cross section;

u means universal for the rising term (independent of projectile hadron). This reduces the number of parameters by 3 units. Assuming again the same factorisation for all pomeron singularities, we get 6 parameters for PL_u, and 7 for $\mathrm{PL2}_u$;

nf means that we have not imposed factorization for the residues of $\mathrm{H}^{ab}(s)$ in the case of the $\gamma\gamma$ and γp cross sections. This adds one parameter to the fit in the case of a single pomeron singularity, and two or three for multiple singularities.

qc means that approximate quark counting rules of the additive quark model [19] are imposed on the residues. This means that the u, d and s couplings can be deduced from pp, πp and Kp scattering, and used to predict Σp. Hence this lowers the number of parameters by 1 unit per singularity to which this rule is applied. It should be noted that analogous counting rules also follow from the so-called gluon dominance model [20] for the dominant asymptotic contribution to the cross sections. These counting rules were confirmed to some extent recently in the global fits of [8].

Finally, we have sometimes assumed that the ratio of the residues of different singularities is process-independent. This is noted by including these singularities in braces {}. We have

also considered the possibility that factorisation works for the lower $C = +1$ trajectories, with the same δ as for the pomeron. We indicate this by putting the singularities in brackets [].

All reasonable combinations of these constraints give more than 256 different variants of the parametrisations. We shall consider here only seven representative models that give a χ^2/dof smaller than 1.5 for all considered energies. Further results may be found in Appendices 1 and 2.

Table 1 gives the results for the minimum center-of-mass energy considered in the fit $\sqrt{s_{min}} = 3$ GeV. Note that because of the large number of points, slight deviations of the χ^2/dof from 1 result in a very low confidence level. Hence we have shown the area of applicability of the models as the energy values for which $\chi^2/dof \leq 1.0$.

	$\sqrt{s_{min}}$ in GeV (number of points)							
Model code (N_{par})	3 (725)	4 (580)	5 (506)	6 (433)	7 (368)	8 (330)	9 (284)	10 (229)
$RRE_{nf}(19)$	1.38	1.15	**0.91**	**0.87**	**0.89**	**0.90**	**0.93**	**0.91**
$RRE^{qc}(17)$	1.39	1.17	**0.93**	**0.89**	**0.90**	**0.91**	**0.93**	**0.92**
$RRL_{nf}(19)$	1.31	**0.96**	**0.82**	**0.80**	**0.85**	**0.85**	**0.86**	**0.85**
RRPL(21)	1.33	**0.98**	**0.85**	**0.83**	**0.87**	**0.88**	**0.84**	**0.74**
$(RR)^d$ P_{nf} L2(20)	1.24	**0.99**	**0.82**	**0.79**	**0.83**	**0.84**	**0.83**	**0.73**
RRP_{nf} $L2_u(21)$	1.26	**0.97**	**0.81**	**0.79**	**0.82**	**0.83**	**0.82**	**0.75**
$(RR)^d$ P $L2_u(17)$	1.28	**1.0**	**0.82**	**0.81**	**0.83**	**0.83**	**0.83**	**0.76**

Table 1: the χ^2/dof of best models fitting all cross section data down to 4 GeV. Numbers in bold represent the area of applicability of each model. In parenthesis, we indicate the number of parameters (N_{par}) for each model.

As can be seen, the data are compatible with many possibilities, and one cannot decide at this level what the nature of the pomeron is, and whether any of the regularities considered above is realised. Note that 9 (resp. 23) models shown in Appendix 1 fit the data well (*i.e.* with a $\chi^2/dof < 1$) for $\sqrt{s_{min}} = 4$ GeV (resp. 5 GeV). Hence it seems that sub-leading trajectories and other non-asymptotic characteristics do not manifest themselves yet. One can see that the logarithmic increases in general fit better than simple powers, even at large energy $\sqrt{s} \sim 10$ GeV, but that the difference in χ^2/dof is not large enough to reach any firm conclusion. Quark counting can be implemented for each possible rising term, but on the other hand one can choose a universal (beam-independent) rise as well. It is interesting

that a reasonable degeneracy of the leading reggeon trajectories can be implemented only in models which have a $\ln^2 s$ pomeron. The latter degeneracy is in fact expected to hold in global fits to the forward scattering data of all hadronic processes, when one includes K^+p scattering, which has an exotic s-channel in view of duality.

We can choose two approaches to distinguish further amongst the above models. We can add more data, which we shall do in the next section, but we want first to examine in detail the quality of the fits. Indeed, despite the fact that these models do fit the data well *globally,* several other characteristics may be considered, and demanded on the results. We shall present here a set of indicators which quantify several aspects of the fits, and which will enable us to assess better the quality of the models.

2.1 Indicators measuring the quality of the fits.

The best known such quantity is certainly the χ^2/dof, or more precisely the confidence level (CL).

However, because Regge theory does not apply in the resonance region, no model is expected to reproduce the data down to the lowest measured energy. The cutoff we have given in Table 1 is *ad hoc*: we know the fits must fail at some point, but we cannot predict where. Hence another indicator will be the *range* of energy of the data that the model can reproduce with a $\chi^2/dof \leq 1.0$.

Furthermore, the quality of the data varies depending on which quantity or which process one considers. In principle, one could introduce some kind of data selection, but that would undoubtedly bias the fits one way or the other. The other option is to assign a weight to each process or quantity, which takes into account the quality of the data. Given that this will be done to compare models together, we are certainly entitled to choose the weights as determined by the best fit. Hence we introduce

$$w_j = \min\left(1, \frac{1}{\chi^2_j/nop}\right)$$

where $j = 1, \ldots 9$ refers to the process, and we define the renormalised χ^2_R as:

$$\chi^2_R \equiv \sum_j w_j \chi^2_j$$

Finally, if a fit is physical in a given range, then its parameters must be stable if one considers part of the range: different determinations based on a sub-sample must be

compatible. Hence another indicator will deal with the *stability* of the fit.

We have developed a series of statistical quantities that enable us to measure the above features of the fits. All these indicators are constructed so that the higher their value the better is the quality of the data description.

(1) The Applicability Indicator: It characterizes the range of energy which can be fitted by the model. This range can in principle be process-dependent, but we shall not consider such a case here. The range of applicability is, by definition, the range of data where fit has a confidence level bigger CL $> 50\%$. One of the simplest variant is as follows:

$$A_j^M = w_j \ln(E_j^{M,high}/E_j^{M,low}), \qquad A^M = \frac{1}{N_{sets}} \sum_j A_j^M \tag{2}$$

where j is the multi-index denoting the pair (data subset, observable); N_{sets} is the number of such subsets, $E_j^{M,high}$ is the highest value of the energy in the area of applicability of the model M in the data subset j; $E_j^{M,low}$ is the lowest value of the energy in the area of applicability of the model M in the data subset j, and w_j is the weight determined from the best fit in the same interval (hence w_j will depend itself on $E_j^{M,high}$ and $E_j^{M,low}$). In our case the applicability indicator takes the form:

$$\begin{aligned} A^M = \ & \tfrac{1}{9} \Big(A^M_{pp,\sigma} + A^M_{\overline{p}p,\sigma} + A^M_{\pi^+ p,\sigma} + A^M_{\pi^- p,\sigma} + A^M_{K^+ p,\sigma} + A^M_{K^- p,\sigma} + \\ & + \ A^M_{\Sigma^- p,\sigma} + A^M_{\gamma p,\sigma} + A^M_{\gamma\gamma,\sigma} \Big) . \end{aligned} \tag{3}$$

Inspection of the fit results shows that for some modification of the parametrisations we obtain rather good fits starting from $E_{min} = 4$ or 5 GeV but with negative contributions to the total cross sections from terms corresponding to the exchange of the pomeron-like objects at low energy part of the area of applicability as defined above. This is unphysical and we are forced to add an additional constraint to the area of applicability: We exclude from it the low energy part where at least in one collision there is a negative contribution from the total sum of the pomeron-like (asymptotically rising) terms. The situation is illustrated in Tables A1.3 and A2.3 of the appendices where excluded intervals are marked by minus as upper case index at the χ^2/dof value. It is interesting that some models turned out to have an empty area of applicability once this criterion was imposed.

(2) Confidence-1 Indicator.

$$C_1^M = CL\%$$

where the CL refers to the whole area of applicability of the model M.

(3) Confidence-2 Indicator.

$$C_2^M = CL\%$$

where the CL refers to the intersection of the areas of applicability of all models qualified for the comparison (we choose here $\sqrt{s} \geq 5$ GeV for the fits without ρ parameter (see Table A1.3) and $\sqrt{s} \geq 9$ GeV for the fits with ρ data (see Table A2.3).

(4) Uniformity Indicator. This indicator measures the variation of the χ^2/nop from bin to bin for some data binning motivated by physics:

$$U^M = \left\{ \frac{1}{N_{sets}} \sum_j \frac{1}{4} \left[\frac{\chi_R^2(t)}{N_{nop}^t} - \frac{\chi_R^2(j)}{N_{nop}^j} \right]^2 \right\}^{-1}, \tag{4}$$

where t denotes the total area of applicability, j is a multi-index denoting the pair (data set, observable). In our case we use the calculation of the χ_R^2/nop for each collision separately, i.e. the sum runs as in the case of the applicability indicator.

(5) Rigidity Indicator. As the measure of the rigidity of the model we propose to use the indicator

$$R_1^M = \frac{N_{dp}^M(A)}{1 + N_{par}^M} \tag{5}$$

The most rigid model has the highest value of the number of data points per adjustable parameter. The exact theory T (with no adjustable parameters) has the rigidity value $R^T = N_{dp}^M(A)$ — the total number of data points in the area of applicability. This indicator takes into account the set of known regularities in the data that were incorporated into the model to reduce the number of adjustable parameters and to increase the statistical reliability of the parameter estimates.

(6) Reliability Indicator.

$$R_2^M = \frac{2}{N_{par}(N_{par}-1)} \cdot \sum_{i>j=1}^{N} \Theta(90.0 - C_{ij}^R) \tag{6}$$

where C_{ij}^R – is the correlation matrix element in % calculated in the fit at the low edge of the applicability area. This indicator characterizes the goodness of the parameter error matrix. For the diagonal correlator this indicator is maximal and equals 1.

(7) Stability-1 Indicator.

$$S_1^M = \left\{ \frac{1}{N_{steps} N_{par}^M} \sum_{steps} \sum_{ij} (P^t - P^{step})_i (W^t + W^{step})_{ij}^{-1} (P^t - P^{step})_j \right\}^{-1} \tag{7}$$

where: P^t - vector of parameters values obtained from the model fit to the whole area of applicability;

P^{step} - vector of parameters values obtained from the model fit to the reduced data set on the *step*, in our case *step* means shift in the low edge of the fit interval to the right by 1 GeV, if there are no steps then $S_1^M = 0$ by definition;

W^t and W^{step} are the error matrix estimates obtained from the fits to the total and to the reduced on the step s data samples from the domain of applicability.

We give the results of these comparisons in Table 2 and Appendix 1, Table A1.2.

The development of these indicators is needed to allow us to verify automatically the rough features of a large quantity of models (see Appendices 1 and 2). Hence, as a first "numerical trigger" to indicate the best fits, we have adopted a simple ranking scheme, which complements the usual "best χ^2" criterion. As all the features measured by the indicators are highly desirable, we adopt for the rank, in a given ensemble of models, a definition that gives equal weight to all indicators

$$I_k^m = (A^m, C_1^m, C_2^m, U^m, R_1^m, R_2^m, S_1^m) \tag{8}$$

where the index m describes the model, index k describes the indicator type.

Having calculated all components of the indicators, it is easy, for a given indicator, to

assign a number of points to a given model M:

$$P_k^M = \sum_{m \neq M} (2\Theta(I_k^M - I_k^m) + \delta_{I_k^M, I_k^m}), \tag{9}$$

The rank of models is then obtained via the total amount of points of the model :

$$P^M = \sum_k P_k^M = \sum_k \sum_{m \neq M} (2\Theta(I_k^M - I_k^m) + \delta_{I_k^M, I_k^m}) \tag{10}$$

In this approach, the best models are the models with the highest P^M value. In the Tables 2 and 5, and in the appendices, we present the ranking of 33 recently discussed parametrisations: 28 of them had a sufficiently high CL for comparison on the σ_{tot}-data and 21 of them had a sufficiently high CL for comparison on the $\sigma_{tot}(s)$- and $\rho(s)$-data.

Model Code	A^M	C_1^M	C_2^M	U^M	R_1^M	R_2^M	S_1^M	rank P^M
simple pole:								
RR $E_{nf}(19)$	**2.6**	91.	81.	51.	25.	0.88	**0.18**	208
RR $E^{qc}(17)$	**2.6**	86.	79.	88.	28.	**0.94**	0.15	**252**
simple+double pole								
$RRL_{nf}(19)$	**2.6**	76.	95.	36.	**29.**	0.79	**0.16**	212
RRPL(21)	2.2	65.	99.7	59.	26.	0.81	0.082	162
simple+triple pole								
$(RR)^d$ $P_{nf}L2(20)$	2.5	59.	**99.9**	38.	28.	0.88	0.098	120
$RRP_{nf}L2_u(21)$	2.5	68.	99.7	34.	26.	0.91	0.008	182
$(RR)^d$ $PL2_u(17)$	**2.6**	**99.8**	99.7	**185.**	28.	0.88	**0.16**	**296**

Table 2: model quality indicators for the models kept in Table 1. Bold-faced characters indicate the best model for a given indicator.

On the other hand, it is also possible to use these indicators directly, as characterizing each model. For instance, if we analyse the first two lines of Table 2, we directly see from column 1 that simple-pole models apply in as big an energy band as the other models. The second and third columns tell us however that the best CL are achieved by triple-pole models with the double-pole models closely behind. The fourth column tells us that while most models do not reproduce all data equally well (see also Table 4), the most uniform model is $(RR)_dPL2_u(17)$. The fifth column indicates that the models apply in similar

energy ranges and have similar numbers of parameters. Similarly, we see from the sixth column that the reliability of the error matrices is similar. However, the seventh column clearly indicates that the parameters of RRPL(21), $(\mathrm{RR})^d$ $\mathrm{P}_{nf}\mathrm{L2}(20)$ and $\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$ are very sensitive to the minimum energy considered, and hence that these models are not stable w.r.t. that minimum energy.

3 Fits to all lower-energy forward data

Given that the fits to total cross sections are unable to decide on the singularity structure of the amplitudes, one can turn to other data, namely the real part of the forward amplitude. It can be obtained through analyticity and $s \to$ crossing symmetry from the form of the cross section (see Appendix 3). If one keeps the same minimum energy, then a joint fit to both cross sections and real parts reaches a very different conclusion. We show in Table 3 the models which achieve a χ^2/dof less than 1 for $\sqrt{s} \geq 5$ GeV.

	$\sqrt{s_{min}}$ in GeV and number of data points							
Model code (N_{par})	3 (904)	4 (742)	5 (648)	6 (569)	7 (498)	8 (453)	9 (397)	10 (329)
$\mathrm{RRE}_{nf}(19)$	1.8	1.4	1.1	1.1	1.1	1.1	1.0	1.0
$\mathrm{RRL}_{nf}(19)$	1.6	1.1	**0.97**	**0.97**	**1.0**	**0.96**	**0.94**	**0.93**
RRPL(21)	1.6	1.1	**0.98**	**0.98**	**0.99**	**0.94**	**0.93**	**0.91**
$(\mathrm{RR})^d$ P_{nf} L2(20)	1.9	1.2	**1.0**	**1.0**	**0.99**	**0.94**	**0.93**	**0.92**
$\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$	1.8	1.1	**0.97**	**0.97**	**0.97**	**0.92**	**0.93**	**0.92**
$(\mathrm{RR})^d$ $\mathrm{PL2}_u(17)$	2.0	1.3	1.0	**1.0**	**0.98**	**0.94**	**0.93**	**0.93**

Table 3: representative models fitting all cross section and ρ data down to 5 GeV. Numbers in bold represent the area of applicability of each model.

The clearest outcome of this is that all models with a simple pole pomeron are then eliminated. The best χ^2/dof for these is 1.12 for RRE_{nf}. Although these values may not seem too problematic, one has to realise that we are fitting to a large number of data points (648 for $\sqrt{s} > 5$ GeV), hence this model is rejected at the 98% C.L.

3.1 Evaluation of the dataset

However, one needs to check where these values of χ^2/dof come from. Hence we can look in detail at the various processes and quantities fitted to. We show in Table 4 the results

of 3 representative models. The first two are kept in Table 3, whereas we came to the conclusion that the third is excluded. We see that the main difference comes from the ρ parameter data, which are much better fitted by the first two models than by the third. However, it is rather difficult to reach a definite conclusion, given the fact that these data are not perfectly fitted by any model: in particular, the πp and pp data.

Reaction	Number of data points	$\mathrm{RRP}_{nf}\mathrm{L2}_{u}$	RRPL	RRE_{nf}
σ_{pp}	112	0.87	0.87	0.89
$\sigma_{\overline{p}p}$	59	1.2	1.0	1.1
σ_{π^+p}	50	0.78	0.78	1.4
σ_{π^-p}	106	0.89	0.90	0.88
σ_{K^+p}	40	0.71	0.72	1.0
σ_{K^-p}	63	0.61	0.62	0.72
σ_{Σ^-p}	9	0.38	0.38	0.39
$\sigma_{\gamma p}$	38	0.62	0.75	0.59
$\sigma_{\gamma\gamma}$	30	0.7	0.95	0.55
ρ_{pp}	74	1.8	1.6	1.8
$\rho_{\overline{p}p}$	11	0.55	0.47	0.60
ρ_{π^+p}	8	1.5	1.6	2.7
ρ_{π^-p}	30	1.2	1.3	2.1
ρ_{K^+p}	10	1.0	1.1	0.83
ρ_{K^-p}	8	0.96	1.2	1.8

Table 4: The values of the χ^2 per data point (χ^2/nop) for each process in three representative models, for $\sqrt{s} > 5$ GeV.

3.2 Best models for all forward data

We can generalize the previous quality indicators to the full set of forward data. We give in Table 5 and in Appendix 2 the quality indicators for representative models fitting both total cross sections and ρ parameters. We have introduced a second stability indicator, S_2,

which is analogous to the stability-1 indicator

$$S_2^M = \left\{ \frac{1}{2N_{par}^M} \sum_{ij} (P^t - P^{t(no\,\rho)})_i (W^t + W^{t(no\,\rho)})^{-1}_{ij} (P^t - P^{t(no\,\rho)})_j \right\}^{-1} . \qquad (11)$$

In this case, we fit the whole set of the model parameters to the full area of applicability (superscript t) and the same set of parameters but to the data sample without ρ-data (superscript $t(no\ \rho)$). This indicator characterizes the reproducibility of the parameters values when fitting to the reduced data sample and reduced number of observables but with the same number of adjustable parameters. This indicator might be strongly correlated with the uniformity indicators. We add S_2^m to the list of indicators entering I_k^m in Eq. (8) when we determine the best models for the full set of data, and run the sums for all indicators for 15 sets of data instead of 9, as we now include the real parts of pp, $\bar{p}p$, $K^{\pm}p$ and $\pi^{\pm}p$.

As we can see, the two parametrisations based on double poles and on triple poles achieve comparable levels of quality, and one cannot decide which is the best based on these indicators. In the conclusion, we shall explain which physics arguments lead us to prefer the triple pole alternative.

Model Code	A^M	C_1^M	C_2^M	U^M	R_1^M	R_2^M	S_1^M	S_2^M	rank P^M
RRP$_{nf}$L2$_u$(21)	**2.2**	68.	**85.**	**23.**	29.	**0.90**	0.22	0.10	**222**
(RR)^d P$_{nf}$L2(20)	**2.2**	50.	82.	18.	31.	**0.90**	0.27	0.41	178
(RR)^d PL2$_u$(17)	2.0	50.	**83.**	16.	**32.**	0.88	**0.30**	0.67	174
RRL$_{nf}$(19)	1.8	**73.**	81.	17.	**32.**	0.78	**0.29**	**1.3**	**222**
RRPL(21)	1.6	**67.**	82.	**26.**	29.	0.75	0.21	**1.1**	173

Table 5: Quality indicators in five representative models fitting well all forward data.

4 Other models

We have tried to impose the Johnson-Treiman-Freund [21, 22] relation for the cross section differences $\Delta\sigma(N) = 5\Delta\sigma(\pi), \Delta\sigma(K) = 2\Delta\sigma(\pi)$, and the models corresponding to this are marked by an index c in Appendices 1 and 2. These rules, while not being totally excluded, never lead to an improvement of the fit, and in some case degrade the fit considerably. It is interesting to note however that they produce the two parametrisations with fewest parameters acceptable above 8 GeV.

We also considered alternative models which have been proposed or rediscovered recently [23, 24], and confront them with our full dataset. From Table 6, one sees clearly that the parameter values and possibly the model themselves have practically zero confidence levels at all starting collision energies $\sqrt{s_{min}}$ from 3 to 10 GeV.

	$\sqrt{s_{min}}$ in GeV						
Model code (N_{par})	3	4	5	6	7	8	9
FFP-97[23]	101	16.26	3.28	2.3	2.3	2.39	2.34
Lipkin TCP[24]	4.63	3.14	2.54	2.61	2.86	3.07	3.48

Table 6: χ^2/dof of two excluded parametrisations.

5 Other data

As in the previous studies [8] of fitting the data sample [12], we have also excluded all cosmic data points [25], [26] in this study of the analytic amplitude models. There are two reasons for that: the original numerical Akeno (Agasa) data are not available and there are the contradictory statements [27, 28, 29, 30, 31]concerning the cross section values of the cosmic data points from both Fly's Eye and Akeno(Agasa).

Having selected the models which reproduce best the accelerator data, we are now able to clarify how well they meet the three cosmic rays data samples. For each cosmic data samples, i.e. those of the original experiments [25], [26]; those corrected by Nikolaev et al. [28], [29]; and those corrected by Block et al. [31] (see also [27]), we calculate the χ^2/nop for each model with parameters fixed at the beginning of their areas of applicability defined by accelerator data. The results are shown in Table 7.

It turns out that the original cosmic experimental data are best fitted by our high-rank parametrisations. The data sample corrected by Block et al. data is also fitted well, as the data points were lowered within the limits of the uncertainties reported in the original experimental publications.

	Experiment		Nikolaev et al.		Block et al.	
Model Code	χ^2	χ^2/nop	χ^2	χ^2/nop	χ^2	χ^2/nop
$\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$	1.62	0.23	14.31	2.04	3.30	0.47
$(\mathrm{RR})^d\mathrm{P}_{nf}\mathrm{L2}_u(19)$	1.73	0.25	13.96	1.99	3.45	0.49
$\mathrm{RRL}_{nf}(19)$	2.52	0.36	24.25	3.46	2.19	0.31
RRPL(21)	2.93	0.42	25.48	3.64	2.34	0.33

Table 7: the χ^2 of the cosmic ray data, corrected in several different ways [27, 28, 29, 30, 31], for each of the best parametrisations fitting the accelerator data.

6 Analysis and conclusion

The above analysis shows that there are several scenarios which can account for the observed forward hadronic scattering amplitudes. These scenarios all have their merits, and some of them have problems. Although only preliminary conclusions can be drawn based on these data, we can outline these various possibilities, and present their consequences.

6.1 Possible parametrisations

The three possible scenarios consist of simple, double or triple poles in the complex J plane accounting for the rising part of the cross section. We give in Table 8 the parameters of each model. All have the same parametrisation for the exchange of the leading meson trajectories, but the values of the various intercepts and residues are very different. The $C = -1$ part of the amplitude is rather stable, but the $C = +1$ part turns out to be very model-dependent as it mixes with the pomeron contribution, with in some cases much larger values of the intercept α_1 than those normally expected from duality-breaking in strong interaction physics. Because of this, the lower energy data cannot fix the nature of the pomeron as the details of the a/f contribution are not known. The data for Σp scattering sometimes lead to a negative a/f contribution, which is incompatible with Regge theory, and to an extrapolation at high energy that overshoots the pp and $\overline{p}p$ cross sections. However, the size of the error bars clearly shows that acceptable values are allowed and that these data do not introduce much of a constraint on the fit.

6.1.1 Simple poles

The first scenario is the simplest conceptually: the pomeron would correspond to some glueball trajectory, and have properties similar to those of the mesons. This model has the advantage that it must then factorize, and hence it can be generalized easily and successfully to many other processes. The residues of the pomeron can also be made totally compatible with quark counting.

It provides good fits to all data for $\sqrt{s} \geq 9$ GeV, acceptable fits for the total cross sections for $\sqrt{s} \geq 5$ GeV, but fails to reproduce both the total cross section and the ρ parameter for $\sqrt{s} \geq 5$ GeV. One can of course take the attitude that the data have problems, and not include them, or that there are sub-dominant effects at these energies, and that it is natural for the model not to be extended so low. On the other side of the energy spectrum, one expects to have unitarity corrections at very large energies. In practice, however, this model differs by a few percents from the RRPL2$_u$ parametrisation, mentioned below, up to LHC energies, and hence unitarizing corrections do not need to be introduced yet.

This model shows a non-degeneracy of the dominant meson trajectories, with somewhat larger a/f intercept α_1 and somewhat smaller ρ/ω intercept α_2, which may well be compatible with the known trajectories.

Furthermore, it is well known that one needs to introduce a new simple pole to account for DIS data in such a scenario. Such a new rising term seems to be totally absent from the soft data, which seems rather odd, but cannot be ruled out. We give in Table 8, column 3, the best parameters for this model in the fit to total cross sections.

6.1.2 Double poles

One can also assume that the amplitude contains a double pole at $J = 1$. This then provides for a rising $\ln s$ term in the total cross section, as well as a constant term. This kind of parametrisation (shown in Table 8, column 2) gives excellent fits to the soft data, and can be extended to deep-inelastic scattering [18] without any further singularity. Furthermore, it never violates unitarity, and hence it can be extended to arbitrarily large energies.

However, it suffers from several drawbacks. First of all, the pomeron term becomes negative below 9.5 GeV, and hence processes which couple only to the pomeron by Zweig's rule would have negative cross sections if one uses factorization. However, the latter is proven only for simple poles, and hence this problem is not a sufficient reason to reject

these parametrisations. Similarly, the split of the leading meson trajectories is quite big, somewhat bigger than what a normal duality-breaking estimate or a linear extrapolation of the known resonances would allow [32]. As a result, the pomeron in this class of variants is inevitably compromising with the crossing even reggeon in the Regge region in the sense that it must effectively counter-balance the excessive contribution of the reggeon. Thus the pomeron term in this case may be representing more than the asymptotic behaviour of the amplitude. One may therefore say that a pomeron associated with reasonably degenerate reggeons may be more natural from the point of view of duality. But again, one cannot prove linearity of the trajectories, hence the model may survive. Finally, it seems that quark counting is respected to a very good approximation by the coefficients of the log and of the constant term. This only reinforces the problem of negativity as it is very difficult to conceive a non factorizing pole which would nevertheless respect quark counting.

6.1.3 Triple poles

Finally, the best fits are given by models that contain a triple pole at $J = 1$, which then produce $\ln^2 s$, $\ln s$ and constant terms in the total cross section. The best parameter values for this model are given in Table 8, column 1. The most interesting properties may be that the constant term respect quark counting to a good approximation, whereas the $\ln^2 s$ term can be taken as universal, i.e. independent of the process, as advocated in [16, 17] and rediscovered in [15] (see also [33]). The universality of the rising term is expected in the case of the eikonal unitarisation of a bare pomeron with the intercept larger than 1, because the coefficient of the rising term turns out to depend only on the intercept and slope of the bare pomeron [34]. But for the J-plane singularities of double and triple pole types considered in this paper, the structure of such a singularity [35] and the origin of its universality is less obvious. Nevertheless, such a singularity at $J = 1$ may in fact have a theoretical explanation: recently, Bartels, Lipatov and Vacca [36] discovered that there are, in fact, two types of Pomeron in LLA : besides the well-known BFKL pomeron associated with 2-gluon exchanges, and with an intercept bigger than 1, there is a second one associated with $C = +1$ three-gluon exchanges and having an intercept precisely located at 1. It is tempting to speculate that, after unitarisation is performed in the gluon sector, the BFKL pomeron would finally lead to a universal Heisenberg-type pomeron, exclusively connected with the gluon sector.

Furthermore, the degeneracy of the lower trajectories is respected to a very good approximation, and the model seems extendible to deep inelastic scattering [37]. This model

also respect unitarity by construction.

One must note that in some processes, the falling $\ln^2(s/s_0)$ term from the triple pole at $s < s_0$ is important in restoring the degeneracy of the lower trajectories at low energy. Hence the squared logarithm manifests itself not only at very high energies, but also at energies below its zero.

Hence we feel that this solution is the one that currently meets all phenomenological and theoretical requirements.

Model	$\mathrm{RRP}_{nf}\mathrm{L2}_u$		RRL_{nf}			RRE_{nf}		
χ^2/dof	0.97		0.97			1.12		
CL[%]	67.98		73.37			2.08		
Parameter	Mean	Uncertainty	Param.	Mean	Uncert.	Param.	Mean	Uncert.
s_0	34.0	5.4	A	-30.3	3.6	$\alpha_{\wp}$	1.0959	0.0021
α_1	0.533	0.015	α_1	0.7912	0.0080	α_1	0.6354	0.0095
α_2	0.4602	0.0064	α_2	0.4555	0.0063	α_2	0.4420	0.0099
Z^{pp}	35.83	0.40	B	6.71	0.22	X^{pp}	18.45	0.41
$Z^{\pi p}$	21.23	0.33	$\lambda_{\pi p}$	0.6833	0.0045	$X^{\pi p}$	11.74	0.24
Z^{Kp}	18.23	0.30	λ_{Kp}	0.6429	0.0073	X^{Kp}	10.45	0.19
$Z^{\Sigma p}$	35.6	1.4	$\lambda_{\Sigma p}$	1.059	0.056	$X^{\Sigma p}$	18.44	1.1
$Z^{\gamma p}$	29.4	3.0	$\lambda_{\gamma p}$	0.00356	0.000048	$X^{\gamma p}$	0.0592	0.0012
$Z^{\gamma\gamma}$	20.4	5.0	$\lambda_{\gamma\gamma}$	$9.37\ 10^{-6}$	$5.2\ 10^{-7}$	$X^{\gamma\gamma}$	0.0001619	$9.7\ 10^{-6}$
Y_1^{pp}	42.1	1.3	Y_1^{pp}	105.8	2.9	Y_1^{pp}	66.1	1.2
Y_2^{pp}	32.19	0.94	Y_2^{pp}	33.36	0.96	Y_2^{pp}	35.3	1.6
$Y_1^{\pi p}$	17.8	1.1	$Y_1^{\pi p}$	60.9	2.4	$Y_1^{\pi p}$	29.40	0.37
$Y_2^{\pi p}$	5.72	0.16	$Y_2^{\pi p}$	5.79	0.16	$Y_2^{\pi p}$	6.04	0.26
Y_1^{Kp}	5.72	1.40	Y_1^{Kp}	49.3	2.5	Y_1^{Kp}	16.43	0.33
Y_2^{Kp}	13.13	0.38	Y_2^{Kp}	13.42	0.38	Y_2^{Kp}	14.07	0.62
$Y_1^{\Sigma p}$	-250.	130.	$Y_1^{\Sigma p}$	82.4	6.4	$Y_1^{\Sigma p}$	-6.	35.
$Y_2^{\Sigma p}$	-320.	150.	$Y_2^{\Sigma p}$	10.	22.	$Y_2^{\Sigma p}$	72.	67.
$Y_1^{\gamma p}$	0.0339	0.0079	$Y_1^{\gamma p}$	0.292	0.013	$Y_1^{\gamma p}$	0.1187	0.0047
$Y_1^{\gamma\gamma}$	0.00028	0.00015	$Y_1^{\gamma\gamma}$	0.000814	0.000040	$Y_1^{\gamma\gamma}$	0.00036	0.00010
δ	0.00371	0.00035						
B	0.3152	0.0095						

Table 8: parameters of three representative models, defined as in Eq. (1), for $\sqrt{s} > 5$ GeV.

6.2 Future prospects

One problem remaining in the analysis of the forward data is the difficulty in adequately fitting the data for the ρ parameter in pp and in $\pi^+ p$ reactions. The extraction of the ρ data from the measurements of the differential cross sections data at small t is a delicate problem. A re-analysis of these data may be needed, but it will call for simultaneous fits to the total cross section data and to the elastic differential cross sections in the Coulomb-nuclear interference region and in the diffractive cones, hence an extension of the parametrisation considered here to the non-forward region. One could also consider a class of analytic models not incorporated in our fits and ranking procedures, class in which the rising terms would turn on at some dynamical threshold s_t (demanding the use of exact dispersion relations), or add lower trajectories to the existing models. Both approaches would lead to many extra parameters, and will be the subject of a future study.

On the other hand, the inclusion of other data may very well allow one to decide finally amongst the various possibilities. One can go to deep-inelastic data, but the problem here is that the photon occupies a special position in Regge theory, and hence the singularities of DIS amplitudes do not need to be the same as those of hadronic amplitudes. One can also extend the models to non-forward data and off-diagonal amplitude such as those of diffractive scattering. Such steps will involve new parameters associated mainly with form factors, but there are many data, hence there is the hope that this kind of systematic study may be generalized, and that in the future we may decide on the nature of Regge singularities.

Finally, it is our intention to develop the ranking scheme further, probably along the lines of [38], and to fine-tune the definition of indicators, in order that a periodic cross assessments of data and models be available to the community [39].

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[39] See the preliminary version of a Web interface at the address http://sirius.ihep.su/~kuyanov/OK/eng/intro.html. It is planned to make the detailed numerical data resulting from the fits easily available on the Web in a

computer-readable form. Meanwhile these data can be obtained by request from tkachenkon@mx.ihep.su (with CC to: kuyanov@mx.ihep.su,ezhela@mx.ihep.su).

Appendix 1. Fits to total cross sections only

In this appendix, we present the results for fits to total cross sections for 33 models, which are variations on the parametrisations referred to in the main text, following the convention explained after Eq. 1. Table A1.1 gives our results for the ranking of the models, according to Eq. 10. Table A1.2 gives the values of the quality indicators associated with each model. Table A1.3 shows the values of the χ^2/dof as a function of energy. The value with a $-$ exponent indicates that the model has a negative pomeron contribution in the low-energy region of the fit. The models marked with * indicates that the extrapolation of the Σp cross sections overshoot the pp or go below $\pi + p$, or that $C = +1$ residues are negative.

Model Code	P_{A^M}	$P_{C_1^M}$	$P_{C_2^M}$	P_{U^M}	$P_{R_1^M}$	$P_{R_2^M}$	$P_{S_1^M}$	Rank P^M
RRL2^{qc}(17)	54	50	18	56	30	50	40	298
(RR)^d PL2$_u$(17)	46	58	46	58	30	24	34	296
(RR$_c$)^d PL2$_u$(15)	30	42	54	54	46	22	46	294
[R^{qc}L2^{qc}]R$_c$(12)	14	44	14	50	52	46	58	278
RRL2(18)	52	54	16	44	18	38	44	266
(RR)^d P^{qc} L2$_u$(16)	28	52	22	46	38	36	42	264
(RR$_c$)^d P^{qc} L2$_u$(14)	18	26	30	40	55	34	52	255
RRE^{qc}(17)	50	36	8	48	30	50	30	252
RR$_c$ L^{qc}(15)	24	32	34	32	46	5	54	227
RR$_c$ E^{qc}(15)	22	38	10	52	20	57	22	221
RR$_c$PL(19)	4	56	56	42	4	0	56	218
[R^{qc} L^{qc}] R(14)	12	48	24	36	55	10	28	213
RRL$_{nf}$(19)*	57	28	36	10	35	14	32	212
RRL^{qc}(17)	57	8	32	26	50	16	20	209
RRE$_{nf}$(19)	48	40	12	30	10	30	38	208
RR$_c$ L2^{qc}(15)	32	0	20	4	46	57	48	207
RRL2$_{nf}$(19)	44	34	6	20	10	54	26	194
(RR)$^d P_{nf} L2$(20)*	42	4	58	16	24	32	18	194
RRPE$_u$(19)	26	46	44	28	10	27	12	193
RRPL2$_u$(19)*	36	14	42	14	35	41	8	190
(RR)^d P$_{nf}$L2$_u$(19)	40	2	48	22	24	27	24	187
RRP$_{nf}$L2$_u$(21)	38	24	51	6	15	44	4	182
[R^{qc} L^{qc}] R$_c$(12)	16	16	26	8	58	5	50	179
(RR)^d {PL2}$_{nf}$(18)	20	18	4	38	6	50	36	172
RRPL(21)*	8	20	51	34	15	18	16	162
RRL(18)	34	10	28	18	41	8	14	153
(RR)^d PL(19)	0	12	0	0	0	41	1	54
(RR)^d P$_{nf}$L$_u$(18)	2	22	2	2	2	20	1	51

Table A1.1: ranking of the 28 models having nonzero area of applicability amongst the 33 in this paper, following Eq. (10), when only total cross sections are fitted to.

Quality indicators							
Model Code	A^M	C_1^M	C_2^M	U^M	R_1^M	R_2^M	S_1^M
$\mathrm{RRL}_{nf}(19)^*$	2.60148	75.54	94.64	35.50	29.05	0.789	0.156
$\mathrm{RRL}^{qc}(17)$	2.60148	59.26	94.09	49.68	32.28	0.794	0.099
$\mathrm{RRL2}^{qc}(17)$	2.58120	97.36	87.91	131.7	28.17	0.941	0.184
RRL2(18)	2.58067	97.52	87.00	85.08	26.68	0.902	0.198
$\mathrm{RR\ E}^{qc}(17)$	2.56576	86.15	79.29	88.38	28.17	0.941	0.146
$\mathrm{RR\ E}_{nf}(19)$	2.56568	91.45	80.78	51.16	25.35	0.883	0.177
$(\mathrm{RR})^d\ \mathrm{PL2}_u(17)$	2.55303	99.78	99.67	184.6	28.17	0.875	0.161
$\mathrm{RRL2}_{nf}(19)$	2.54792	81.62	77.64	41.85	25.35	0.942	0.143
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}(20)^*$	2.53820	58.94	99.88	37.60	27.67	0.884	0.098
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}_u(19)$	2.53154	54.71	99.72	44.31	27.67	0.877	0.114
$\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$	2.52375	67.76	99.73	34.40	26.41	0.910	0.008
$\mathrm{RRPL2}_u(19)^*$	2.52351	62.59	99.65	37.14	29.05	0.906	0.018
RRL(18)	2.52103	59.95	93.52	39.85	30.58	0.693	0.068
$\mathrm{RR}_c\ \mathrm{L2}^{qc}(15)$	2.50642	54.11	88.31	26.54	31.69	0.952	0.259
$(\mathrm{RR}_c)^d\ \mathrm{P\ L2}_u(15)$	2.47739	94.20	99.75	97.71	31.69	0.838	0.220
$(\mathrm{RR})^d\ \mathrm{P}^{qc}\ \mathrm{L2}_u(16)$	2.46789	97.49	92.53	87.39	29.82	0.900	0.197
$\mathrm{RRPE}_u(19)$	2.44915	95.83	99.66	49.82	25.35	0.877	0.057
$\mathrm{RR}_c\ \mathrm{L}^{qc}(15)$	2.42625	78.91	94.41	51.99	31.69	0.667	0.331
$\mathrm{RR}_c\ \mathrm{E}^{qc}(15)$	2.39977	89.51	79.88	95.81	27.13	0.952	0.104
$(\mathrm{RR})^d\{\mathrm{PL2}\}_{nf}(18)$	2.39430	64.63	72.14	70.24	22.84	0.941	0.164
$(\mathrm{RR}_c)^d\ \mathrm{P}^{qc}\ \mathrm{L2}_u(14)$	2.38295	75.32	93.62	74.78	33.80	0.890	0.310
$[\mathrm{R}^{qc}\ \mathrm{L}^{qc}]\ \mathrm{R}_c$	2.37016	63.32	92.89	34.57	39.00	0.667	0.289
$[\mathrm{R}^{qc}\ \mathrm{L2}^{qc}]\ \mathrm{R}_c(12)$	2.36985	94.28	83.36	91.56	33.38	0.924	0.491
$[\mathrm{R}^{qc}\ \mathrm{L}^{qc}]\ \mathrm{R}(14)$	2.36207	96.86	92.55	59.94	33.80	0.736	0.145
$\mathrm{RRPL}(21)^*$	2.18238	64.98	99.73	58.88	26.41	0.810	0.082
$\mathrm{RR}_c\mathrm{PL}(19)$	1.93416	99.20	99.84	78.20	21.70	0.561	0.372
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L}_u(18)$	1.62709	65.46	65.46	14.98	12.11	0.810	0.000
$(\mathrm{RR})^d\ \mathrm{PL}(19)$	1.40760	62.17	62.17	14.83	11.50	0.906	0.000

Table A1.2: quality indicators of the the 28 models having nonzero area of applicability amongst the 33 models considered in this paper, following Eqs. (2-7) when only total cross sections are fitted to.

χ^2/dof vs. $\sqrt{s_{min}}$ in GeV								
ModelCode(N_{par})	3	4	5	6	7	8	9	10
$RRE_{nf}(19)$	1.38	1.15	**0.91**	**0.87**	**0.89**	**0.90**	**0.93**	**0.91**
$RRE^{qc}(17)$	1.39	1.17	**0.93**	**0.89**	**0.90**	**0.91**	**0.93**	**0.92**
$RR_c\ E^{qc}(15)$	2.37	1.47	1.05	**0.91**	**0.90**	**0.91**	**0.93**	**0.91**
$RRL_{nf}(19)^*$	1.31	**0.96**$^-$	**0.82**	**0.80**	**0.85**	**0.85**	**0.86**	**0.85**
RRL(18)	1.33	**0.98**	**0.85**	**0.83**	**0.87**	**0.87**	**0.87**	**0.86**
$RRL^{qc}(17)$	1.33	**0.99**$^-$	**0.85**	**0.83**	**0.87**	**0.87**	**0.87**	**0.85**
$RR_c\ L^{qc}(15)$	2.20	1.22	**0.95**$^-$	**0.84**	**0.86**	**0.86**	**0.87**	**0.85**
$[R^{qc}\ L^{qc}]\ R(14)$	1.44	1.03	**0.88**$^-$	**0.85**	**0.89**	**0.87**	**0.88**	**0.87**
$[R^{qc}\ L^{qc}]\ R_c(12)$	2.20	1.22	**0.95**$^-$	**0.84**	0.86	0.86	**0.87**	**0.85**
$RRL2_{nf}(19)$	1.45	1.19	**0.94**	**0.90**	**0.91**	**0.91**	**0.94**	**0.92**
RRL2(18)	1.33	1.05	**0.88**	**0.85**	0.91	0.89	0.90	0.89
$RRL2^{qc}(17)$	1.33	1.06	**0.88**	**0.85**	0.88	0.88	0.90	0.89
$RR_c\ L2^{qc}(15)$	2.28	1.33	**0.99**	**0.87**	**0.87**	**0.88**	**0.90**	**0.89**
$[R^{qc}\ L2^{qc}]\ R_c(12)$	2.39	1.38	1.03	**0.89**	**0.90**	**0.89**	**0.91**	**0.91**
$(RR)^d\ L^{qc}(15)$	2.63	2.02	1.37	1.27	1.22	1.21	1.25	1.08
$(RR)^d\ PL(19)$	2.34	1.84	1.34	1.24	1.21	1.21	1.22	**0.97**
$(RR)^d\ P^{qc}\ E_u(16)$	1.44	1.16	1.02	1.01	1.06	1.06	1.05	1.04
$(RR)^d\{PL2\}_{nf}(18)$	1.91	1.56	1.06	**0.97**	**0.95**	**0.95**	**0.99**	**0.94**
$RRPL(21)^*$	1.33	**0.98**$^-$	**0.85**$^-$	**0.83**$^-$	**0.87**	**0.88**	**0.84**	**0.74**
$RR_cPL(19)$	1.33	**0.98**$^-$	**0.85**$^-$	**0.83**$^-$	**0.87**$^-$	**0.87**$^-$	**0.84**	**0.74**
$RRPL_{u,nf}(20)^-$	2.24	1.42	1.14	1.03	**0.97**$^-$	**0.91**$^-$	**0.84**$^-$	**0.74**$^-$
$RRPL_u(18)^-$	2.24	1.43	1.16	1.05	**0.99**$^-$	**0.93**$^-$	**0.85**$^-$	**0.76**$^-$
$(RR)^d\ P_{nf}L_u(18)$	2.66	2.10	1.73	1.58	1.43	1.37	1.25	**0.96**
$(RR)^d\ P^{qc}\ L_u(15)$	2.74	2.27	2.06	2.06	2.12	2.15	2.19	2.38
$(RR)^d\ P_{nf}\ L2(20)^*$	1.24	**0.99**	**0.82**	**0.79**	**0.83**	**0.84**	**0.83**	**0.73**
$RRPL2_u(21)$	1.26	**0.97**	**0.81**	**0.79**	**0.82**	**0.83**	**0.82**	**0.75**
$RRPL2_u(19)^*$	1.27	**0.98**	**0.82**	**0.80**	**0.84**	**0.84**	**0.83**	**0.76**
$(RR)^d\ P_{nf}\ L2_u(19)$	1.27	**0.99**	**0.82**	**0.80**	**0.83**	**0.83**	**0.82**	**0.75**
$(RR)^d\ P\ L2_u(17)$	1.28	1.00	**0.82**	**0.81**	**0.83**	**0.83**	**0.83**	**0.76**
$(RR)^d\ P^{qc}\ L2_u(16)$	1.30	1.04	**0.88**	**0.87**	**0.91**	**0.91**	**0.90**	**0.86**
$(RR_c)^d\ P\ L2_u(15)$	2.08	1.19	**0.90**	**0.82**	**0.83**	**0.83**	**0.82**	**0.75**
$(RR_c)^d\ P^{qc}\ L2_u(14)$	2.11	1.22	**0.96**	**0.88**	**0.90**	**0.90**	**0.89**	**0.86**
$RRPE_u(19)$	1.36	1.04	**0.89**	**0.86**	**0.87**	**0.86**	**0.83**	**0.76**

Table A1.3: χ^2/dof as a function of the minimum energy of the fit for the 33 models considered in this paper when only total cross sections are fitted to.

Appendix 2. Fits to total cross sections and to the ρ parameter.

In this appendix, we present the results for fits to total cross sections and the ρ parameter for 33 models, which are variations on the parametrisations referred to in the main text, following the convention explained after Eq. 1. Only 21 of these passed through qualification tests in this case. The tables are presented as in Appendix 1. It should be noted that for model RRPL2$_u$(19) with highest rank, corresponding to model RRP$_{nf}$L2$_u$(21) with the extra imposition of factorization on the P_{ab} residues, tends to choose a negative value for the reggeon $C = +1$ residue in $\gamma\gamma$ cross sections. Although this does not exclude it as the residue has large errors, we have preferred to present in this paper the details of the next best ranking parametrisation.

Model Code	P_{A^M}	$P_{C_1^M}$	$P_{C_2^M}$	P_{U^M}	$P_{R_1^M}$	$P_{R_2^M}$	$P_{S_1^M}$	$P_{S_2^M}$	Rank P^M
$\mathrm{RRPL2}_u(19)^*$	42	26	42	42	34	28	12	4	230
$\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$	44	36	44	40	15	31	10	2	222
$\mathrm{RRL}_{nf}(19)^*$	30	42	26	24	34	18	18	30	222
$(\mathrm{RR}_c)^d\ \mathrm{PL2}_u(15)$	34	20	36	20	28	24	28	14	204
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}_u(19)$	40	8	40	22	34	22	16	12	194
$[\mathrm{R}^{qc}\ \mathrm{L}^{qc}]\mathrm{R}_c(12)$	14	32	18	10	42	6	24	38	184
$(\mathrm{RR}_c)^d\mathrm{P}^{qc}\mathrm{L2}_u(14)$	20	16	10	36	19	36	22	22	181
$(\mathrm{RR})^d\mathrm{P}^{qc}\mathrm{L2}_u(16)$	18	14	8	38	8	38	30	26	180
$\mathrm{RR}_c\ \mathrm{L2}^{qc}(15)$	6	30	6	4	6	44	44	40	180
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}(20)^*$	38	2	28	32	25	31	14	8	178
$(\mathrm{RR})^d\ \mathrm{PL2}_u(17)$	36	0	34	18	30	26	20	10	174
$\mathrm{RRPL}(21)^*$	2	34	32	44	15	16	6	24	173
$\mathrm{RR}_c\ \mathrm{L}^{qc}(15)$	24	38	24	8	10	4	32	32	172
$\mathrm{RRL2}^{qc}(17)$	10	28	4	2	2	42	40	42	170
$[\mathrm{R}^{qc}\ \mathrm{L2}^{qc}]\mathrm{R}_c(12)$	12	18	0	6	22	40	38	34	170
$\mathrm{RRL}^{qc}(17)$	28	6	20	30	44	12	4	18	162
$\mathrm{RRPE}_u(19)$	22	44	12	16	4	20	34	6	158
$[\mathrm{R}^{qc}\mathrm{L}^{qc}]\mathrm{R}(14)$	16	24	14	12	19	14	36	20	155
RRL2(18)	8	22	2	0	0	34	42	44	152
$\mathrm{RR}_c\mathrm{PL}(19)$	4	12	38	14	12	0	26	36	142
RRL(18)	26	10	16	26	39	8	8	0	133

Table A2.1: ranking of the the 21 models having nonzero area of applicability amongst the 33 models considered in this paper, following Eq. (10) when cross sections and ρ parameters are fitted to.

Quality indicators								
Model Code	A^M	C_1^M	C_2^M	U^M	R_1^M	R_2^M	S_1^M	S_2^M
$\mathrm{RRP}_{nf}\mathrm{L2}_u(21)$	2.20661	67.98	84.74	22.88	29.45	0.900	0.224	0.101
$\mathrm{RRPL2}_u(19)^*$	2.20619	63.46	84.13	24.14	32.40	0.895	0.226	0.190
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}_u(19)$	2.18781	53.15	83.81	16.49	32.40	0.871	0.286	0.690
$(\mathrm{RR})^d\ \mathrm{P}_{nf}\mathrm{L2}(20)^*$	2.18530	50.41	81.74	18.21	30.86	0.900	0.265	0.407
$(\mathrm{RR})^d\ \mathrm{PL2}_u(17)$	1.99653	50.35	83.04	15.64	31.61	0.882	0.296	0.673
$(\mathrm{RR}_c)^d\ \mathrm{PL2}_u(15)$	1.88491	61.92	83.38	16.26	31.13	0.876	0.467	0.795
$\mathrm{RRL}_{nf}(19)^*$	1.82464	73.37	81.09	16.63	32.40	0.784	0.289	1.302
$\mathrm{RRL}^{qc}(17)$	1.82281	52.97	78.17	17.56	36.00	0.743	0.198	1.080
RRL(18)	1.82274	53.59	77.18	16.73	34.11	0.686	0.217	0.001
$\mathrm{RR}_c\ \mathrm{L}^{qc}(15)$	1.82270	68.31	79.68	12.48	28.31	0.667	0.525	1.311
$\mathrm{RRPE}_u(19)$	1.81878	73.98	73.74	15.46	22.65	0.830	0.526	0.282
$(\mathrm{RR}_c)^d\mathrm{P}^{qc}\mathrm{L2}_u(14)$	1.79558	60.29	67.08	19.94	30.20	0.912	0.429	1.100
$(\mathrm{RR})^d\mathrm{P}^{qc}\mathrm{L2}_u(16)$	1.79315	58.40	66.41	19.98	26.65	0.917	0.470	1.241
$[\mathrm{R}^{qc}\mathrm{L}^{qc}]\mathrm{R}(14)$	1.73409	63.29	76.41	13.09	30.20	0.747	0.533	1.082
$[\mathrm{R}^{qc}\ \mathrm{L}^{qc}]\mathrm{R}_c(12)$	1.73264	65.79	78.13	13.03	34.85	0.682	0.440	1.935
$[\mathrm{R}^{qc}\ \mathrm{L2}^{qc}]\ \mathrm{R}_c(12)$	1.72644	61.50	61.50	11.58	30.54	0.939	1.159	1.692
$\mathrm{RRL2}^{qc}(17)$	1.72618	64.20	64.20	11.23	22.06	0.941	1.318	2.503
RRL2(18)	1.72607	63.04	63.04	11.19	20.89	0.902	1.395	2.657
$\mathrm{RR}_c\ \mathrm{L2}^{qc}(15)$	1.72369	65.63	65.63	11.27	24.81	0.952	1.447	2.104
$\mathrm{RR}_c\mathrm{PL}(19)$	1.99062	55.13	83.67	15.38	28.45	0.61	0.466	1.824
$\mathrm{RRPL}(21)^*$	1.60724	66.59	82.16	26.29	29.45	0.752	0.210	1.135

Table A2.2: quality indicators of the 21 models having nonzero area of applicability amongst the 33 models considered in this paper, following Eqs. (2-7) and (11) when cross sections and ρ parameters are fitted to.

χ^2/dof vs. $\sqrt{s_{min}}$ in GeV								
Model Code(N_{par})	3	4	5	6	7	8	9	10
$RRE_{nf}(19)$	1.83	1.38	1.12	1.10	1.10	1.05	1.02	1.02
$RRE^{qc}(17)$	1.84	1.39	1.13	1.12	1.11	1.06	1.02	1.02
$RR_c\ E^{qc}(15)$	2.47	1.58	1.23	1.13	1.10	1.05	1.02	1.02
$RRL_{nf}(19)^*$	1.61	1.10	**0.97**$^-$	**0.97**$^-$	1.00	**0.96**	**0.94**	**0.93**
RRL(18)	1.63	1.13	**0.99**$^-$	**0.99**$^-$	1.02	**0.97**	**0.95**	**0.94**
$RRL^{qc}(17)$	1.63	1.13	**1.00**$^-$	**0.99**$^-$	1.02	**0.97**	**0.94**	**0.94**
$RR_c\ L^{qc}(15)$	2.20	1.30	1.08	1.01	1.02	**0.97**	**0.94**	**0.94**
$[R^{qc}\ L^{qc}]\ R(14)$	1.70	1.16	1.02	1.01	1.03	**0.98**$^-$	**0.95**	**0.94**
$[R^{qc}\ L^{qc}]\ R_c(12)$	2.20	1.30	1.08	1.01	1.02	**0.97**$^-$	**0.94**	**0.94**
$RRL2_{nf}(19)$	1.83	1.34	1.11	1.10	1.11	1.06	1.01	1.00
RRL2(18)	1.68	1.22	1.04	1.04	1.06	1.01	**0.97**	**0.97**
$RRL2^{qc}(17)$	1.68	1.22	1.04	1.04	1.05	1.01	**0.97**	**0.97**
$RR_c\ L2^{qc}(15)$	2.30	1.41	1.13	1.06	1.05	1.00	**0.97**	**0.97**
$[R^{qc}\ L2^{qc}]\ R_c(12)$	2.38	1.44	1.16	1.07	1.07	1.01	**0.98**	**0.98**
$(RR)^d\ L^{qc}(15)$	3.76	2.61	1.87	1.82	1.73	1.70	1.72	1.72
$(RR)^d\ PL(19)$	3.45	2.37	1.81	1.76	1.71	1.69	1.73	1.72
$(RR)^d\ P^{qc}\ E_u(16)$	2.35	1.53	1.24	1.23	1.21	1.17	1.17	1.17
$(RR)^d\{PL2\}_{nf}(18)$	2.81	1.98	1.40	1.34	1.27	1.20	1.13	1.12
$RRPL(21)^*$	1.63	1.11	**0.98**$^-$	**0.98**$^-$	**0.99**$^-$	**0.94**$^-$	**0.93**$^-$	**0.91**
$RR_cPL(19)$	1.63	1.11	**0.98**$^-$	**0.98**$^-$	**0.99**$^-$	**0.94**$^-$	**0.93**$^-$	**0.91**
$RRPL_{u,nf}(20)^-$	2.43	1.49	1.25	1.16	1.08	1.00$^-$	**0.97**$^-$	**0.92**$^-$
$RRPL_u(18)^-$	2.43	1.50	1.27	1.17	1.10	1.01	**0.98**$^-$	**0.93**$^-$
$(RR)^d\ P_{nf}L_u(18)$	3.59	2.50	2.10	1.95	1.91	1.88	1.89	1.87
$(RR)^d\ P^{qc}\ L_u(15)$	3.67	2.64	2.32	2.27	2.32	2.32	2.39	2.51
$(RR)^d\ P_{nf}\ L2(20)^*$	1.92	1.23	**1.00**	**1.00**	**0.99**	**0.94**	**0.93**	**0.92**
$RRP_{nf}L2_u(21)$	1.75	1.14	**0.97**	**0.97**	**0.97**	**0.92**	**0.93**	**0.92**
$RRPL2_u(19)^*$	1.75	1.15	**0.98**	**0.98**	**0.97**	**0.93**	**0.93**	**0.92**
$(RR)^d\ P_{nf}\ L2_u(19)$	1.96	1.26	**0.99**	**0.99**	**0.98**	**0.93**	**0.93**	**0.93**
$(RR)^d\ PL2_u(17)$	1.96	1.27	1.00	**1.00**	**0.98**	**0.94**	**0.93**	**0.93**
$(RR)^d\ P^{qc}\ L2_u(16)$	1.98	1.29	1.04	1.04	1.03	**0.98**	**0.97**	**0.97**
$(RR_c)^d\ PL2_u(15)$	2.38	1.37	1.06	1.01	**0.98**	**0.93**	**0.93**	**0.93**
$(RR_c)^d\ P^{qc}\ L2_u(14)$	2.40	1.39	1.10	1.05	1.03	**0.98**	**0.97**	**0.97**
$RRPE_u(19)$	1.88	1.22	1.06	1.03	1.01	**0.96**	**0.95**	**0.93**

Table A2.3: χ^2/dof as a function of the minimum energy of the fit for the 33 models considered in this paper when cross sections and ρ parameters are fitted to.

Appendix 3. Formulæ

We give here the formulae used in this paper. The imaginary part of the amplitude, which we take as s time the total cross section, is parametrized as the sum of several terms, I_n, with (see Eq. (1)):

$$I^+_{pole} = C^+(s/s_1)^{\alpha_+} \tag{12}$$

$$I^-_{pole} = \mp C^-(s/s_1)^{\alpha_-} \tag{13}$$

$$I_L = C_L\ s\ln(s/s_1) \tag{14}$$

$$I_{L2} = C_{L2}\ s\ln^2(s/s_0) \tag{15}$$

All terms have charge conjugation $C = +1$, except I^-_{pole} which has $C = -1$. We can obtain the corresponding additive real parts through s to u crossing symmetry and analyticity:

$$R^+_{pole} = -I^+_{pole}\cot\left[\frac{\pi}{2}\ \alpha_+\right] \tag{16}$$

$$R^-_{pole} = I^-_{pole}\tan\left[\frac{\pi}{2}\ \alpha_-\right] \tag{17}$$

$$R_L = \frac{\pi}{2}\ s\ C_L \tag{18}$$

$$R_{L2} = \pi\ s\ \ln(s/s_0)\ C_{L2} \tag{19}$$